**Topics: Normal distribution, Functions of Random Variables**

* The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

1. 0.3875
2. 0.2676
3. 0.5
4. 0.6987

ANS:

Normal Distribution *µ* = 45 minutes

*σ* = 8 minutes

*Sample data x=60-10=50*

Standard Normal Distribution Z=x- *µ* /*σ*

*There ,*

*P(x>50),*

*Z=50-45/8*

*=0.625*

*Z table,*

*P(x>50),*

*Z=0.73237=73.23%*

*For,*

*P(x>50)=1-P(x>50)*

*=1-Z*

*=1-73237*

*=0.23763*

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| *ANS:B* |

*,*

* The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.

1. More employees at the processing center are older than 44 than between 38 and 44

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

ANS:

N=400, *σ* =6,*μ* = 38.

a)

TO find ,

P(x>44) and P(x<38)=?

P(x>44),

*Z1=44-38/6*

*=1*

*Z Table,*

*Z1 =0.84134*

*For,*

*P(x>44)=1-P(x>44)*

*=1-Z1*

*=1-0.84133*

*=0.15866*

P(x<38*),*

*Z2=38-38/6*

*=0*

*Z Table,*

*Z2=0.5*

*For,*

*P(x>38)=1-P(x>38)*

*=1-Z2*

*=1-0.5*

*=0.5*

P(38<x>44)= P(x>44) - P(x<38)

=*0.15866-0.5*

*=0.34%*

*Therefore,*

P(38<x>44) is value 0.34%

The processing center are older than 44 than between 38 and 44 is True

b)

P(x>30),

*Z=30-38/6=-1.3333*

*Z Table,*

*Z=0.09176=9.17%*

*For,*

*Same employees probability is 0.9171 so be under*

*n=400,*

*30=400\*0.09176=36.68=36*

The training program for employees under the age of 30 at the center would be expected to attract about 36 employees is True

* If *X1*~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

ANS:

Given:

*X1*~ *N*(μ, σ2)

*X*2 ~ *N*(μ, σ2)

To find:

2 *X*1 and *X*1 + *X*2?

\* Distributions 2X1 \* Distributions *X*1 + *X*2

plot line(X1)=(1,2,3,4,5) mean : μ+ μ=2 μ

then, Variance: 1σ2+1σ2=2σ2

2(X1)=(2,4,6,8,10)

Mean: 2\* μ=2μ

Variance: (2σ)2=4σ2

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| ANSWER : **Distributions of the both mean values is same but variance are 2X1 is greater than X1+X2.** |

* Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

1. 90.5, 105.9
2. 80.2, 119.8
3. 22, 78
4. 48.5, 151.5
5. 90.1, 109.9

ANS:

Given:

X ~ N(100, 202),

Therfore;

*μ* = 100, *σ* 2= 400, *σ* = 20

Standard Normal Distribution Z=x- *µ* /*σ*

*X=Z- µ* /*σ=2.576*

*-2.576=*48.5

+2.576=151.5

|  |  |
| --- | --- |
| Answer | D |

* Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

ANS:

Given:

~ N(5, 32),

µ1=5, *σ1=3*

~ N(7, 42) ,

µ2=7, *σ2=4*

*addition information is* $1 = Rs. 45,

*Total mean*  µ= µ1+ µ2=5+7=12

Total variance *σ=σ1+σ2=3+4=7*

Convert the doller to rupees,

µ=45\*12=540

*σ=45\*7=225*

Formula, Z=*/σ (Z0.25=±1.92 )*

Z-0.25==99

Z+0.25=

Range is (99,981)

1. Specify the 5th percentile of profit (in Rupees) for the company

ANS:

Z0.05==169.875 (z0.05=±1.645)

Specify the 5th percentile of profit (in Rupees) for the company=169.875

1. Which of the two divisions has a larger probability of making a loss in a given year?

ANS:

Formula ,

Z=x-µ1/*σ1*

P(d1<0)=0-5/3= -1.66

P(d2<0)=0-7/4= -1.75

**Both are same but distribution 1 are more loss than distribution 2**